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PROBLEMS AND QUESTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

PROBLEMS FOR SOLUTION.

ALGEBRA.

387. Proposed by H. C. FEEMSTER, York, Neb.

Sum the following series:

$$(1) \sum_{n=1}^{n=\infty} \frac{1}{8^{n+1}n!}; \quad (2) \sum_{n=1}^{n=\infty} \frac{1.3.5. \cdots 2n+1}{2^{2n+1}(2n-1)!}; \quad (3) \sum_{n=1}^{n=\infty} \frac{1}{2^{2n}n!}$$

GEOMETRY.

415. Proposed by WILLIAM D. CAIRNS, Oberlin College.

For three points on a straight line, P_1, P_2, P_3 , we have

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}; \quad P_2P_3 = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2};$$

and

$$P_1P_3 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2},$$

while

$$P_1P_2 + P_2P_3 = P_1P_3.$$

Prove directly that the sum of the first two radical expressions equals the third.

416. Proposed by W. H. BUSSEY, University of Minnesota.

Let AB be a diameter of any circle, and let C be the third vertex of the equilateral triangle of which AB is one side. Divide AB into six equal parts and let P denote the second point of division from A (i. e., $AP = \frac{1}{3}AB$). Draw the straight line CP and produce it to meet the circle in R . Prove that AR is one-sixth of the circumference.*

CALCULUS.

337. Proposed by R. P. BAKER, University of Iowa.

Show that for a, b relatively prime integers,

$$\int_0^1 (\cos 2\pi ax + \cos 2\pi bx) dx = \frac{2}{\pi ab} \left[\frac{a+b}{\sin \frac{\pi}{a+b}} - \frac{a-b}{\sin \frac{\pi}{a-b}} \right],$$

or

$$\frac{1}{\pi ab} \left[(a+b) \cot \frac{\pi}{2(a+b)} - (a-b) \cot \frac{\pi}{2(a-b)} \right]$$

according as a and b are both odd or one of them is even.

* *Note of explanation:* This problem was called to my attention by a member of the class in design in our art department. A generalization of the construction can be used to divide a circle approximately into any number of equal parts. If the diameter AB is divided into n equal parts, and if P be the second point of division from A , the line CP produced will meet the circle in R so that the arc AR is an arc of $360^\circ/n$. The construction is theoretically exact when $n = 6$ and in the trivial cases when $n = 2$ or 4 . It is used, I am told, when $n = 5$ and 7 and the construction is very approximately correct. The problem for $n = 6$ is not difficult if one uses analytic geometry or trigonometry. Perhaps some reader of the MONTHLY can work it by means of elementary geometry. I asked the student who called the problem to my attention why she did not use the ordinary geometric constructions for dividing a circle into 5 or 6 equal parts. She said she wanted one construction that would be good in all cases.

338. Proposed by RICHARD LOCHNER, Philadelphia, Pa.

An elliptical field has a major axis of 100 feet and a minor axis of 10 feet. A cow is tethered at the end of the major axis and another at the end of the minor axis. If each cow can graze over half the field, how long is the rope of each? What is the area of the portion over which the cows can graze in common?

MECHANICS.

274. Proposed by G. B. M. ZERR.

A sphere moves on the concave side of a rough cylindrical surface of which the transverse section perpendicular to the generating lines is a hypocycloid. If $s = l \sin n\theta$ be the intrinsic equation of the hypocycloid, then $l = (a - b)4b/a$, $n = a/(a - 2b)$, where a = radius of fixed circle, b = radius of rolling circle.

275. W. J. GREENSTREET, Editor Mathematical Gazette, England.

If a particle be attracted towards the angular points of a regular hexagon by forces equal to r^{-h} , at distance r , find the condition for stability of equilibrium.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

190. Proposed by H. C. FEEMSTER, York, Neb.

Show that, if n is a prime number and r is an integer less than n , then

$$\frac{r-1}{n} \mid \frac{n-r}{n} + (-1)^{r-1} = M \cdot n,$$

where M is an integer.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

376. Proposed by W. W. BEMAN, Ann Arbor, Mich.

If

$$\left(1 + \frac{1}{m}\right)^m / e = 1 - a_1 \frac{1}{m} + a_2 \frac{1}{m^2} - a_3 \frac{1}{m^3} + \dots,$$

prove that

$$na_n = \sum_{k=1}^{k=n} \frac{k}{k+1} a_{n-k}$$

and compute $a_1, a_2, a_3, \dots, a_8$.

SOLUTION BY THE PROPOSER.

For convenience, put $1/m = x$, and $y = (1 + x)^{1/x}/e$. Then

$$ey = (1 + x)^{1/x},$$

$$\log y = -\frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots$$

and

$$\frac{y'}{y} = -\frac{1}{2} + \frac{2x}{3} - \frac{3x^2}{4} - \dots.$$

Assume

$$y = 1 - a_1x + a_2x^2 - a_3x^3 + \dots.$$

Then

$$\begin{aligned} y' &= -a_1 + 2a_2x - 3a_3x^2 + 4a_4x^3 - \dots \\ &= \left(1 - a_1x + a_2x^2 - a_3x^3 + \dots\right) \left(-\frac{1}{2} + \frac{2x}{3} - \frac{3x^2}{4} + \dots\right). \end{aligned}$$